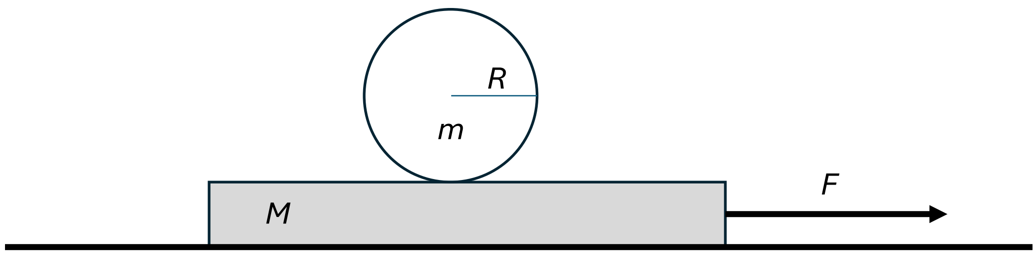
Teacher notes
Topic A

Rolling on an accelerating block

A cylinder of mass m and radius R is on top of a block of mass M. A horizontal force F acts on the block accelerating it to the right. The floor is frictionless. The cylinder rolls on the block without slipping.

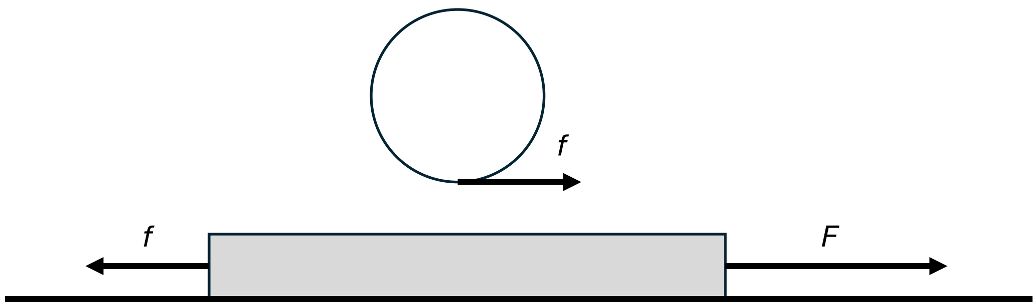


(a) A student considers the cylinder and the block as one body and claims that they have a common acceleration of $\frac{F}{M+m}$. Comment on the student's claim.
(b) Draw free body diagrams for the horizontal forces on the cylinder and the block.
(c) What is the acceleration of the center of mass of the cylinder and of the block relative to an observer on the ground?
(d) If F is the maximum force that allows rolling without slipping for the cylinder, determine the coefficient of static friction between the cylinder and the block.

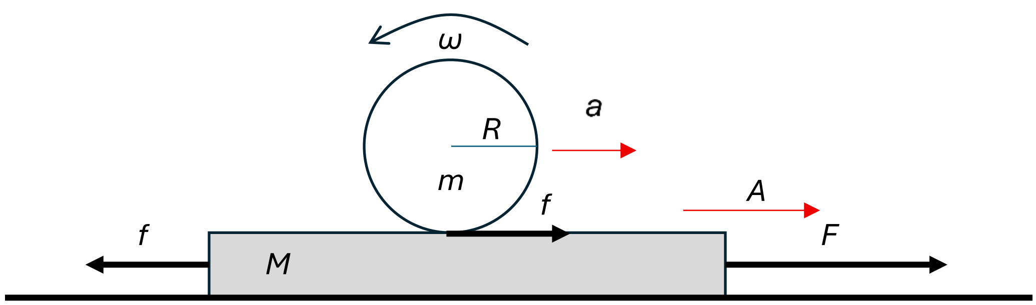
Answers

(a) The student's answer would be correct if the cylinder and the block had the same acceleration. But they do not since the cylinder will move on the block, so the trick of combining the two bodies as one does not work here.

(b) As the block accelerates to the right, the cylinder tends to slide to the left so a frictional force pushes it to the right. By Newton's third law an equal magnitude force acts on the block towards the left.



(c) Let A be the acceleration of the block and a that of the CM of the cylinder both as measured by an inertial observer on the ground. We expect the cylinder to rotate counterclockwise and accelerate to the right relative to the ground hence the following diagram:



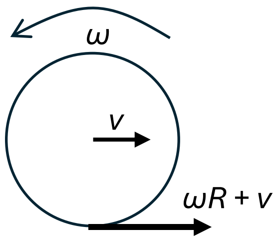
Newton's second law applied to the block gives

$F - f = MA$

Applied to the cylinder it gives

$f = ma$

The diagram shows the velocity of the point of the cylinder in contact with the block. Because we have rolling without slipping: $\omega R + v = V$ where V is the velocity of the block relative to the ground. It follows that $\alpha R + a = A$.



The rotational version of Newton's second law applied to the cylinder gives $fR = I\alpha = I\dfrac{a}{R}$. From $\alpha R + a = A$ we find $\alpha = \dfrac{A-a}{R}$ and so ($I = \dfrac{1}{2}mR^2$)

$fR = \dfrac{1}{2}mR^2\dfrac{A-a}{R} \quad \Rightarrow \quad f = \dfrac{1}{2}m(A-a)$.

Putting this in $f = ma$ gives

$\dfrac{1}{2}m(A-a) = ma \quad \Rightarrow \quad a = \dfrac{A}{3}$

Hence $f = \dfrac{1}{2}m(A - \dfrac{A}{3}) = \dfrac{1}{3}mA$ and so $F = MA + f = MA + \dfrac{mA}{3} = A(M + \dfrac{m}{3})$ and finally $A = \dfrac{F}{M + \dfrac{m}{3}}$ or

$$A = \frac{3F}{3M+m},$$

and

$$a = \frac{F}{3M+m}.$$

(Notice that the acceleration of the cylinder **relative to the block** is $a' = a - A = -\dfrac{2A}{3}$. The cylinder accelerates to the left relative to the block and rotates counterclockwise. The CM accelerates to the right relative to the ground.)

(d) $f = \dfrac{1}{3}mA$ and $f = \mu mg$ so $\mu mg = \dfrac{1}{3}mA$ giving

$$\mu = \frac{A}{3g} = \frac{F}{(3M+m)g}.$$